Written Exam for the B.Sc. or M.Sc. in Economics, Winter 2010/2011

Corporate Finance and Incentives

Final Exam/ Elective Course/ Master's Course

22. February 2011

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Problem 1.

1. Explain the concepts complete and arbitrage free markets. Then answer whether the following market is complete and arbitrage free.

$$C = \begin{pmatrix} 100 & 0 & 0 \\ 5 & 105 & 0 \\ 3 & 3 & 103 \end{pmatrix}, \pi = \begin{pmatrix} 94 \\ 99 \\ 96 \end{pmatrix}$$

C is the cashflow matrix of market and π is the price of the cashflows.

2. Define and write up the APT model. Explain why it can be seen as an extension of the CAPM model.

3. Explain the difference between American and European call options, especially on dividend and non-dividend paying assets/stocks.

4. A stock is currently trading at price 100 and the risk-free interest rate is 5%. An at-the-money (ATM) 1-year call option (i.e. with strike K=100) is trading at price 5, whereas the corresponding ATM put option is trading at price 4. Use the put-call parity, given by $c_o + PV(K) = p_0 + S_0$, where c_0 and p_0 is the price of a call and a put option, PV(K) is the present value of K and S_0 is the share price, to show the existence of an arbitrage opportunity in the market. How would you exploit this arbitrage.

5. Describe the value on a European call option using the Black-Scholes framework (see 1.6 for formulas if needed), when a) volatility increases, b) the stock price increases and c) the time to expiry increases.

6. Draw the the Black-Scholes call price in the strike-interval $K = 80 - 120$ for an option with price S=100, $\sigma = 0.2$, $T = 2$ and $r_f = 5\%$. Explain the intuition behind the graph.

The Black-Scholes formula is given by:

$$c = SN(d_1) - Ke^{-r_f T} N(d_2),$$

where

$$d_1 = \frac{ln(S/K) + \left(r_f + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\sqrt{T}.$$

The $N()$ function is the cumulative probabilities for a unit (zero mean and 1 in variance) normal variable z.

7. An private equity fund uses a standard capital structure of 25% equity and 75% debt in its investments. It operates with a return on equity of 25% and faces debt costs of 10% in the market. Interest rate payments are deductible with a 40% tax rate. What is the companys cost of capital on its investments. Explain the method used.

8. Give at least 3 theoretical reasons/explanations behind the inherent conflict between debtholders and equityholders.

Problem 2.

Consider a bond with the following cash flow:

Year	1	2	3
Bond	4	6	108

and the following zero-coupon yield curve:

Year	1	2	3
Percentage	4.50%	6.00%	7.00%

1. Find the price of the bond and find the price change following from a 1 percentage point increase in the yield curve.

2. Find the yield-to-maturity on the bond. Why is the yield-to-maturity close to the third zero-coupon yield (i.e. 7%). (Note: You do not have the Solver available in Excel, hence an approximate answer will suffice.)

3. Find the duration and convexity of the bond, using the formulas:

$$Dur = \frac{1}{PV\left(C, y\right)} \sum_{t=1}^{T} t \frac{c_t}{\left(1+y\right)^t}$$

and

$$Cvx = \frac{1}{PV\left(C, y\right)} \sum_{t=1}^{T} t^2 \frac{c_t}{\left(1+y\right)^t}$$

PV is the price of the bond, c_t is the cashflow at time t, t is the time period and y is the yield-to-maturity.

Then assume the entire yield curve increases by 1%. Use the duration and convexity measures to approximate the price changes. What explains the difference from the answer in question 1 - is it a good approximation.

4. We now learn that the bond contains an embedded call option, where the issuer of the bond can choose to prepay the bond at price 100 - regardless of its actual market price. Explain verbally, using options theory, how the price of the option and the bond would be affected by changes in the yield curve.

Problem 3.

The Prime Investment Company is an investment company, which has specialized in investments in the 3 assets given in the table below. Currently their, and the market in general, return expectations on a 1-year horizon - dependent on the current state of the economy - is as given in the below table.

1. Calculate the average return and the covariance matrix.

Table 1: The market

	Poor State (33%)	Average State (34%)	Good State (33%)
Risk-free asset	3%	3%	3%
HighQ Asset	1%	4%	12%
MediumQ Asset	0%	11%	7%
LowQ Asset	16%	16%	-12%

2. Find the minimum variance portfolio.

3. Find the tangency portfolio. Sketch, using the two-fund separation theorem, the mean-variance frontier.

4. The Prime Investment Company has been given the mandate to hold a portfolio that is twice as risky, measured by the variance, as the market portfolio. They are allowed to borrow at 4% in the markets. What is the highest archievable expected return on this twice-risky portfolio on a 1-year horizon.